A matter of survival: a simple model for the detection of an invasive species under surveillance.

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Centre of Excellence for Biosecurity Risk Analysis

Incursion Impacts

The impact of an invasion is affected by a range of factors, many of which can be anticipated in advance, for example,

- the prevalence of host material,
- likely climate suitability,
- pathways,
- the value of affected agriculture, and
- so on.

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Size at Detection is Pivotal. Size at Detection cannot be anticipated.

Switch the Flip: Detection is Death

Let's draw an analogy between detection analysis and survival analysis.

Componentry: The Hazard function

Instantaneous probability of detection: given p,

$$h(x, p) = 1 - (1 - p)^x. \qquad (1)$$

1. The only thing that changes is the incursion size x.
2. Here, x is continuous.
3. This function assumes that the incursion has not been detected before it reaches size x.
4. This function assumes that the growth of the incursion is linear in time. We will address this assumption later.
5. This function assumes that the detections of the x pests by the trap network are independent. We will address this assumption in a different presentation with suitable weaponry.

Componentry: pdf of time to detection

The relationship between the hazard and the pdf of time to detection is known.

$$
\begin{aligned}
f(x_d, p) &= h(x_d, p) \times \exp\left(-\int_0^{x_d} h(k, p) dk\right) && (2) \\
&= \{1 - (1-p)^{x_d}\} \times \exp\left(-\int_0^{x_d} 1 - (1-p)^k \, dk\right) && (3) \\
&= \{1 - (1-p)^{x_d}\} \times \exp\left(-\frac{1-(1-p)^{x_d}}{\log(1-p)} - x_d\right). && (4)
\end{aligned}
$$

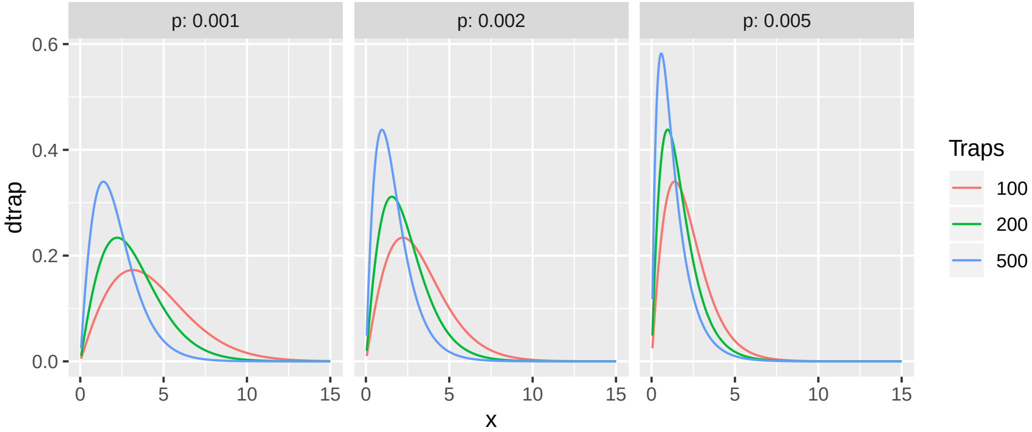
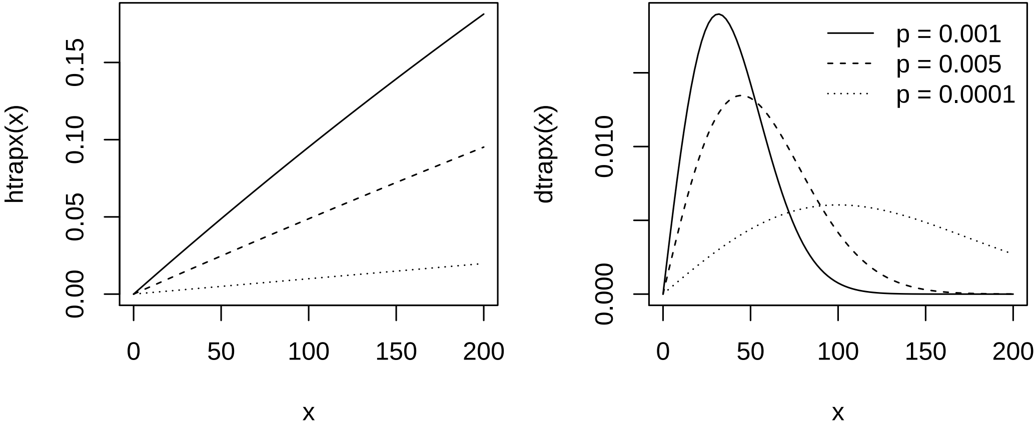


Figure: The pdf for base model with a range of values for t and p.

Sidebar: a Weibull Approximation for Locally Linear Hazard

For small p, $1 - (1-p)^x \approx px$.



Weibull is Everyhere

When the hazard function is exactly straight, then the pdf is known to be a member of the Weibull family with shape parameter 2. This is because the hazard function for the Weibull distribution with shape α and scale β is:

$$h(x, \alpha, \beta) = \frac{\alpha}{\beta^\alpha} x^{\alpha - 1} \tag{5}$$

so

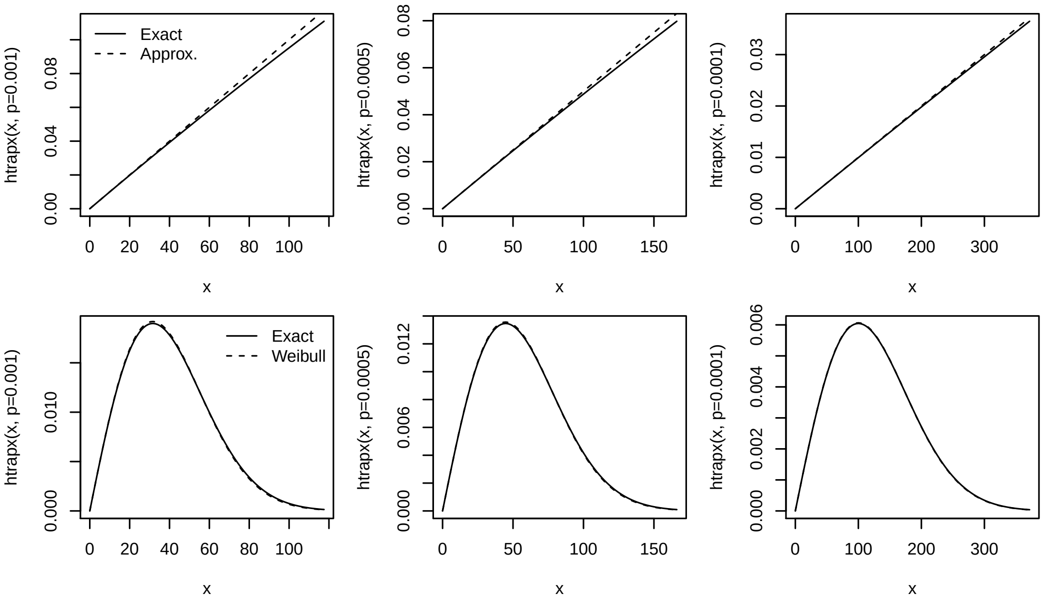
$$h(x, \alpha = 2, \beta) = \frac{2}{\beta^2} x, \tag{6}$$

which is linear in x.

Scale Parameter

We can obtain the approximate slope of the hazard function as follows. The scale parameter is a function of the slope of the hazard function: $2/\beta^2$, so $\beta = \sqrt{2/p}$.

So: for sufficiently low p and the usual probability aggregating function (1), the pdf of the incursion size at the time of detection will be closely approximated by the Weibull distribution with shape parameter $\alpha = 2$ and scale parameter $\beta = \sqrt{2/p}$.



Exact
Approx.
htrapx(x, p=0.001)
htrapx(x, p=0.0005)
htrapx(x, p=0.0001)
x
Exact
Weibull
htrapx(x, p=0.001)
htrapx(x, p=0.0005)
htrapx(x, p=0.0001)
x

Time or Size?

What if growth relative to p is not linear in time?

Change of Variables: briefly, given a pdf $f(x_d)$ and a transformation (i.e., growth model) $y(x_d)$, we evaluate $f(y)$ as $f(x_d(y)) \times dx_d/dy$.

Imagine that the incursion growth is locally quadratic. We need to evaluate the pdf at the points of the inverse of that function (the square root) and then correct it for the dimensional change on the axis (pdfs are obliged to integrate to 1).

$y(x) = x^2$, so $x(y) = \sqrt{y}$ and $dx/dy = 1/(2\sqrt{y})$.

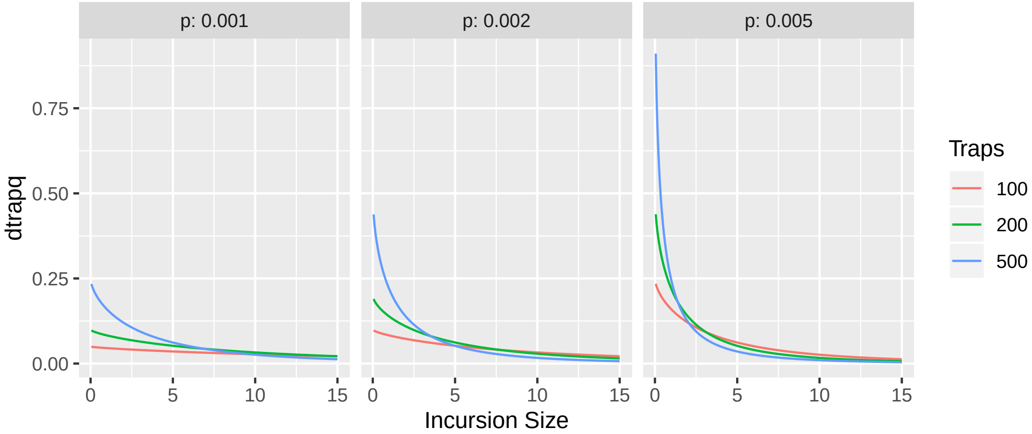
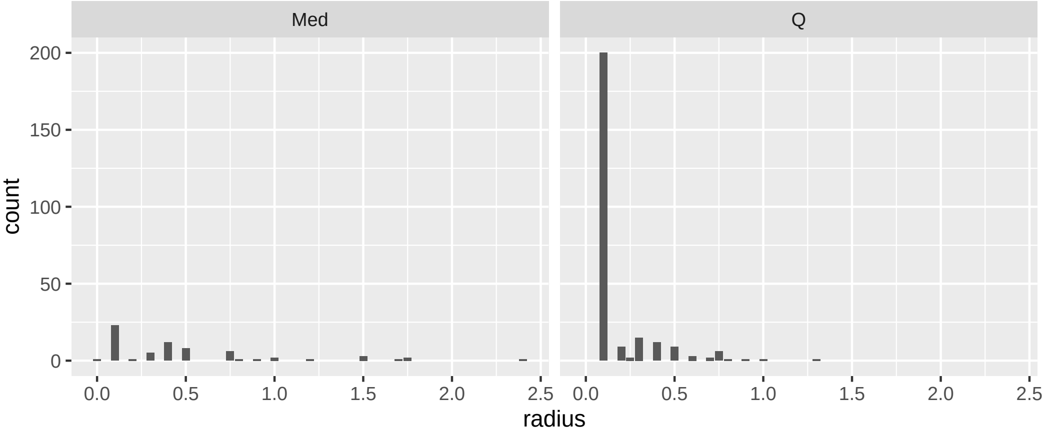


Figure: PDF under quadratic growth with a range of parameter values.

Case Study: QFF/Medfly in South Australia



Data courtesy of Alan Meats.

Wrap Up

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- Mike Ormsby
- Alan Meats

Appendix: Koopman detection

Following Koopman (searching for naval vessels), assume that the probability of detection for a single pest is

$$p(t, s, e, a) = s \times \text{erf}\left(0.855\frac{et}{a}\right) \tag{7}$$

where a is the area of the host material, e is the effective sampling area of a trap, s is the average sensitivity of the trap within the area, and t is the number of traps. Then the hazard function for x pests is, assuming independence of the pest trapping events:

$$h(x, t, s, e, a) = 1 - \left(1 - s \times \text{erf}\left(0.855\frac{et}{a}\right)\right)^x \tag{8}$$

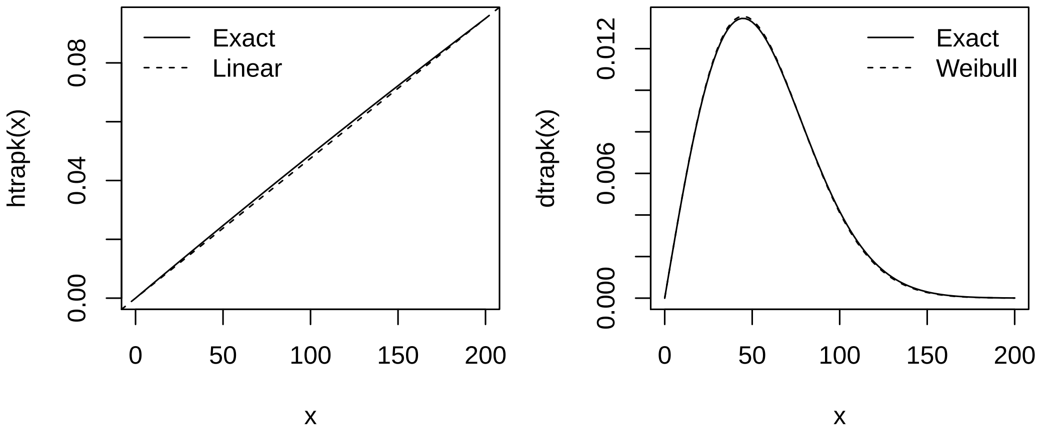
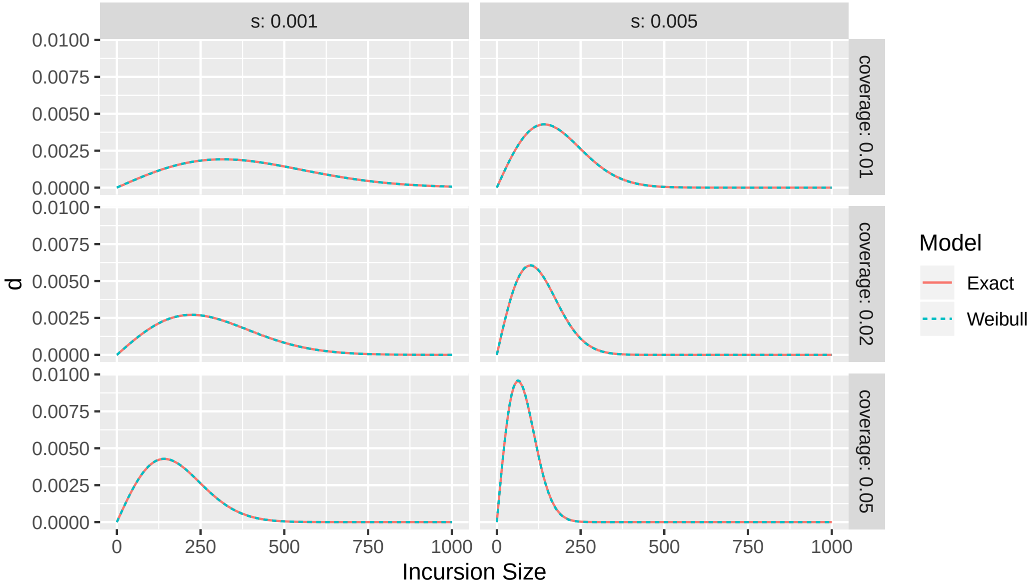


Figure: The hazard and pdf for 500 1-ha traps with sensitivity 1% distributed regularly in 10,000 ha.



s: 0.001
s: 0.005
coverage: 0.01
coverage: 0.02
coverage: 0.05
d
0.0100
0.0075
0.0050
0.0025
0.0000
0
250
500
750
1000
Incursion Size
Model
Exact
Weibull